7 - 15 Evaluation of determinants
Showing the details, evaluate:

7. $\begin{vmatrix} \text{Cos}[\alpha] & \text{Sin}[\alpha] \\ \text{Sin}[\beta] & \text{Cos}[\beta] \end{vmatrix}$

```
ClearAll["Global`*"]
```

```
e1 = Det[( Cos[α]  Sin[α] )
         ( Sin[β]  Cos[β] )]
```

```
Cos[α] Cos[β] - Sin[α] Sin[β]
```

```
e2 = TrigReduce[e1]
```

```
Cos[α + β]
```

The above cell matches the answer in the text.

9. $\begin{vmatrix} \text{Cos}[n\,\theta] & \text{Sin}[n\,\theta] \\ -\text{Sin}[n\,\theta] & \text{Cos}[n\,\theta] \end{vmatrix}$

```
ClearAll["Global`*"]
```

```
e1 = Det[( Cos[n θ]   Sin[n θ] )
         ( -Sin[n θ]  Cos[n θ] )]
```

```
Cos[n θ]^2 + Sin[n θ]^2
```

```
e2 = TrigReduce[e1]
```

```
1
```

The above cell matches the answer in the text.

11. $\begin{vmatrix} 4 & -1 & 8 \\ 0 & 2 & 3 \\ 0 & 0 & 5 \end{vmatrix}$

```
ClearAll["Global`*"]
```

```
Det[( 4  -1  8 )
    ( 0   2  3 )
    ( 0   0  5 )]
```

```
40
```

The above cell matches the answer in the text.

13. $\begin{vmatrix} 0 & 4 & -1 & 5 \\ -4 & 0 & 3 & -2 \\ 1 & -3 & 0 & 1 \\ -5 & 2 & -1 & 0 \end{vmatrix}$

```
ClearAll["Global`*"]
```

```
e1 = Det[{{0, 4, -1, 5}, {-4, 0, 3, -2}, {1, -3, 0, 1}, {-5, 2, -1, 0}}]
```

```
289
```

The above cell matches the answer in the text.

15. $\begin{vmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 2 & 0 \\ 0 & 2 & 9 & 2 \\ 0 & 0 & 2 & 16 \end{vmatrix}$

```
ClearAll["Global`*"]
```

```
e1 = Det[{{1, 2, 0, 0}, {2, 4, 2, 0}, {0, 2, 9, 2}, {0, 0, 2, 16}}]
```

```
-64
```

The above cell matches the answer in the text.

17 - 19 Rank by determinants
Find the rank by theorem 3, p. 297, (which is not very practical) and check by row reduction.

17. $\begin{vmatrix} 4 & 9 \\ -8 & -6 \\ 16 & 12 \end{vmatrix}$

```
ClearAll["Global`*"]
```

```
e1 = {{4, 9}, {-8, -6}, {16, 12}}
```

```
{{4, 9}, {-8, -6}, {16, 12}}
```

```
e1 = MatrixRank[e1]
```

```
2
```

The above cell matches the answer in the text.

19. $\begin{vmatrix} 1 & 5 & 2 & 2 \\ 1 & 3 & 2 & 6 \\ 4 & 0 & 8 & 48 \end{vmatrix}$

```
ClearAll["Global`*"]
```

```
e1 = {{1, 5, 2, 2}, {1, 3, 2, 6}, {4, 0, 8, 48}}
```

$$e1 = \begin{pmatrix} 1 & 5 & 2 & 2 \\ 1 & 3 & 2 & 6 \\ 4 & 0 & 8 & 48 \end{pmatrix}$$

```
e2 = MatrixRank[e1]
```

```
2
```

The above cell matches the answer in the text.

21 - 25 Cramer's rule
Solve by Cramer's rule. Check by Gauss elimination and back substitution.

21. 3 x - 5 y = 15.5
6 x + 16 y = 5.0

```
ClearAll["Global`*"]
```

```
e1 = {{3, -5}, {6, 16}}
```

$$e1 = \begin{pmatrix} 3 & -5 \\ 6 & 16 \end{pmatrix}$$

```
e2 = {15.5, 5.0}
```

```
{15.5, 5.}
```

```
e3 = {x, y}
```

```
{x, y}
```

```
e4 = Thread[e1.e3 == e2]
```

```
{3 x - 5 y == 15.5, 6 x + 16 y == 5.}
```

```
e5 = Solve[e4, e3]
```

```
{{x → 3.5, y → -1.}}
```

```
e6 = e4 /. e5
```

```
{{True, True}}
```

The above cell matches the answer in the text.

23. 3 y - 4 z = 16
2 x - 5 y + 7 z = -27
-x - 9 z = 9

```
ClearAll["Global`*"]
```

```
e1 = {{0, 3, -4}, {2, -5, 7}, {-1, 0, -9}}
```

```
{{0, 3, -4}, {2, -5, 7}, {-1, 0, -9}}
```

```
e2 = {16, -27, 9}
```

```
{16, -27, 9}
```

```
e3 = {x, y, z}
```

```
{x, y, z}
```

```
e4 = Thread[e1.e3 == e2]
```

```
{3 y - 4 z == 16, 2 x - 5 y + 7 z == -27, -x - 9 z == 9}
```

```
e5 = Solve[e4, e3]
```

```
{{x → 0, y → 4, z → -1}}
```

```
e6 = e4 /. e5
```

```
{{True, True, True, True}}
```

The above cell matches the answer in the text.

25. - 4 w + x + y = -10
w - 4 x + z = 1
w - 4 y + z = -7
x + y - 4 z = 10

```
ClearAll["Global`*"]
```

```
e1 = {{-4, 1, 1, 0}, {1, -4, 0, 1}, {1, 0, -4, 1}, {0, 1, 1, -4}}
```

```
{{-4, 1, 1, 0}, {1, -4, 0, 1}, {1, 0, -4, 1}, {0, 1, 1, -4}}
```

```
e2 = {-10, 1, -7, 10}
```

```
{-10, 1, -7, 10}
```

```
e3 = {w, x, y, z}
```

```
{w, x, y, z}
```

```
e4 = Thread[e1.e3 == e2]
```

```
{-4 w + x + y == -10, w - 4 x + z == 1, w - 4 y + z == -7, x + y - 4 z == 10}
```

```
e5 = Solve[e4, e3]
```

```
{{w → 3, x → 0, y → 2, z → -2}}
```

```
e6 = e4 /. e5
```

```
{{True, True, True, True}}
```

The above cell matches the answer in the text.